A.- Hallar las siguientes integrales inmediatas o integrales por cambio de variable:

1) $\int \frac{2}{\sqrt{x}} \cdot dx = 2\int x^{-1/2} \cdot dx = 2 \cdot \frac{x^{1/2}}{1/2} + C = 4\sqrt{x} + C$

2) $\int \frac{(Lnx)^8}{x} \cdot dx = \int (Lnx)^8 \cdot \frac{1}{x} \cdot dx = \frac{(Lnx)^9}{9} + C$

3) $\int \frac{x^2 - x^3 \cdot senx}{x^3} \cdot dx = \int \frac{x^2}{x^3} \cdot dx - \int \frac{x^3 \cdot senx}{x^3} \cdot dx = \int \frac{1}{x} \cdot dx - \int senx \cdot dx = Ln|x| + \cos x + C$

4) $\int \frac{\sqrt[3]{x} + \sqrt{x}}{\sqrt[4]{x}} \cdot dx = \int \frac{x^{1/3}}{x^{1/4}} \cdot dx + \int \frac{x^{1/2}}{x^{1/4}} \cdot dx = \int x^{1/3 - 1/4} \cdot dx + \int x^{1/2 - 1/4} \cdot dx = \int x^{1/12} \cdot dx + \int x^{1/4} \cdot dx =$

$= \frac{x^{13/12}}{13/12} + \frac{x^{5/4}}{5/4} + C = \frac{12 \cdot \sqrt[12]{x^{13}}}{13} + \frac{4 \cdot \sqrt[4]{x^5}}{5} + C$

5) $\int \frac{1+2x}{1+x^2} \cdot dx = \int \frac{1}{1+x^2} \cdot dx + \int \frac{2x}{1+x^2} \cdot dx = \text{arctg}x + \text{Ln}(1+x^2) + C$

6) $\int 5x \cdot \sqrt{x^2-3} \cdot dx = \frac{5}{2} \cdot \int 2x \cdot (x^2-3)^{1/2} \cdot dx = \frac{5}{2} \cdot \frac{(x^2-3)^{3/2}}{3/2} + C = \frac{5 \cdot \sqrt{(x^2-3)^3}}{3} + C$

7) $\int x \cdot (3x^2+5)^{24} \cdot dx = \frac{1}{6} \cdot \int 6x \cdot (3x^2+5)^{24} \cdot dx = \frac{1}{6} \cdot \frac{(3x^2+5)^{25}}{25} + C = \frac{(3x^2+5)^{25}}{150} + C$

8) $\int \frac{1}{(x-4)^5} \cdot dx = \int (x-4)^{-5} \cdot dx = \frac{(x-4)^{-4}}{-4} + C = -\frac{1}{4 \cdot (x-4)^4} + C$

9) $\int \frac{4x^2}{\sqrt{5-x^3}} \cdot dx = \frac{4}{-3} \cdot \int -3x^2 \cdot (5-x^3)^{-1/2} dx = -\frac{4}{3} \cdot \frac{(5-x^3)^{1/2}}{1/2} + C = -\frac{8 \cdot \sqrt{5-x^3}}{3} + C$

10) $\int \frac{\cot gx}{sen^2 x} \cdot dx = \int \cot gx \cdot \frac{1}{sen^2 x} \cdot dx = -\int \cot gx \cdot \frac{-1}{sen^2 x} \cdot dx = -\int t \cdot dt = -\frac{t^2}{2} + C = -\frac{\cot g^2 x}{2} + C$

t = cotgx $\rightarrow$ dt = $-1/sen^2 x \cdot dx$

11) $\int \frac{3x}{x^2+5} \cdot dx = \frac{3}{2} \cdot \int \frac{2x}{x^2+5} \cdot dx = \frac{3}{2} \cdot Ln|x^2+5| + C$

12) $\int \frac{x+1}{x^2+2x+5} \cdot dx = \frac{1}{2} \cdot \int \frac{2x+2}{x^2+2x+5} \cdot dx = \frac{1}{2} \cdot Ln|x^2+2x+5| + C$

13) $\int \frac{5x^2}{sen^2(x^3+5)} \cdot dx = \frac{5}{3} \int \frac{3x^2}{sen^2(x^3+5)} \cdot dx = \frac{5}{3} \cdot (-\cot g(x^3+5)) + C = -\frac{5}{3} \cdot \cot g(x^3+5) + C$

14) $\int 3^{-3x+5} \cdot dx = \frac{1}{-3} \cdot \int 3^{-3x+5} \cdot (-3) \cdot dx = -\frac{1}{3} \cdot \frac{3^{-3x+5}}{Ln3} + C = -\frac{3^{-3x+5}}{3 \cdot Ln3} + C$

15) $\int 4x^2 \cdot e^{5x^3-3} \cdot dx = \frac{4}{15} \int 15x^2 \cdot e^{5x^3-3} \cdot dx = \frac{4}{15} \cdot e^{5x^3-3} + C$

16) $\int \cos(2x^3-5) \cdot 3x^2 \cdot dx = \frac{3}{6} \int \cos(2x^3-5) \cdot 6x^2 \cdot dx = \frac{1}{2} \cdot sen(2x^3-5) + C = \frac{sen(2x^3-5)}{2} + C$

17) $\int (1+tg^2(2x)) \cdot dx = \frac{1}{2} \cdot \int (1+tg^2(2x)) \cdot 2 \cdot dx = \frac{1}{2} \cdot tg(2x) + C = \frac{tg(2x)}{2} + C$

18) $\int tg^2 x \cdot dx = \int \left(tg^2 x + 1 - 1\right) \cdot dx = \int \left(tg^2 x + 1\right) \cdot dx - \int 1 \cdot dx = \text{tgx} - \text{x} + \text{C}$

19) $\int \frac{1}{1+9x^2} \cdot dx = \int \frac{1}{1+(3x)^2} \cdot dx = \frac{1}{3} \cdot \int \frac{3}{1+(3x)^2} \cdot dx = \frac{1}{3} \cdot arctg(3x) + \text{C}$

20) $\int \frac{4}{49+x^2} \cdot dx = \int \frac{4/49}{1+x^2/49} \cdot dx = \frac{4}{49} \cdot 7 \cdot \int \frac{1/7}{1+\left(x/7\right)^2} \cdot dx = \frac{4}{7} \cdot arctg\left(\frac{x}{7}\right) + \text{C}$

21) $\int \frac{1}{\sqrt{9-64x^2}} \cdot dx = \int \frac{1/\sqrt{9}}{\sqrt{1-64x^2/9}} \cdot dx = \frac{1}{8} \int \frac{8/3}{\sqrt{1-\left(8x/3\right)^2}} \cdot dx = \frac{1}{8} \cdot arcsen\left(\frac{8x}{3}\right) + \text{C}$

22) $\int \frac{sen\left(1/x\right)}{x^2} \cdot dx = \int sen\left(1/x\right) \cdot \frac{1}{x^2} \cdot dx = -\int sen\left(1/x\right) \cdot \frac{-1}{x^2} \cdot dx = -\left(-\cos\left(\frac{1}{x}\right)\right) + \text{C} = \cos\left(\frac{1}{x}\right) + \text{C}$

23) $\int \frac{\cos\left(\sqrt{x}\right)}{3 \cdot \sqrt{x}} \cdot dx = \int \cos\left(\sqrt{x}\right) \cdot \frac{1}{3 \cdot \sqrt{x}} \cdot dx = \frac{2}{3} \cdot \int \cos\left(\sqrt{x}\right) \cdot \frac{1}{2 \cdot \sqrt{x}} \cdot dx = \frac{2}{3} \cdot sen\left(\sqrt{x}\right) + \text{C}$

24) $\int \frac{x^2}{1+x^6} \cdot dx = \int \frac{x^2}{1+\left(x^3\right)^2} \cdot dx = \frac{1}{3} \cdot \int \frac{3x^2}{1+\left(x^3\right)^2} \cdot dx = \frac{1}{3} \cdot arctg\left(x^3\right) + \text{C}$

25) $\int \frac{4x}{\cos^2\left(5x^2-1\right)} \cdot dx = \frac{4}{10} \cdot \int \frac{10x}{\cos^2\left(5x^2-1\right)} \cdot dx = \frac{2}{5} \cdot tg\left(5x^2-1\right) + \text{C}$

B.- Calcular las siguientes integrales, por partes:

1) $\int x^2 \cdot senx \cdot dx$ = x·(-cosx) - $\int (-\cos x) \cdot 2x \cdot dx$ = -x·cosx + $\int \cos x \cdot 2x \cdot dx$ = (*)

u = x^2 ⇒ du = 2x·dx ‖ u = 2x ⇒ du = 2·dx

dv = senx·dx ⇒ v = $\int senx \cdot dx$ = - cosx ‖ dv = cosx·dx ⇒ v = $\int \cos x \cdot dx$ = senx

(*)= - x·cosx + [2x·senx - $\int senx \cdot 2 \cdot dx$] = - x·cosx + 2x·senx – 2·$\int senx \cdot dx$ =

= - x·cosx + 2x·senx + 2·cosx + C

2) $\int x^6 \cdot Lnx \cdot dx$ = Lnx·$\frac{x^7}{7}$ - $\int \frac{x^7}{7} \cdot \frac{1}{x} \cdot dx = \frac{x^7 \cdot Lnx}{7} - \frac{1}{7}\int x^6 \cdot dx = \frac{x^7 \cdot Lnx}{7} - \frac{1}{7} \cdot \frac{x^7}{7}$ + C =

u = Lnx ⇒ du = $1/x$·dx ‖ = $\frac{x^7 \cdot Lnx}{7} - \frac{x^7}{49}$ + C

dv = x^6·dx ⇒ v = $\int x^6 \cdot dx = \frac{x^7}{7}$

3) $\int (3x+1) \cdot sen(2x-1) \cdot dx = (3x+1) \cdot \left(-\frac{\cos(2x-1)}{2}\right) - \int \left(-\frac{\cos(2x-1)}{2}\right) \cdot 3 \cdot dx$ = (*)

u = 3x + 1 ⇒ du = 3·dx

dv = sen(2x-1)·dx ⇒ v = $\frac{1}{2} \cdot \int sen(2x-1) \cdot 2 \cdot dx = -\frac{\cos(2x-1)}{2}$

(*) = $-\frac{(3x+1) \cdot \cos(2x-1)}{2} + \frac{3}{2} \cdot \int \cos(2x-1) \cdot dx = -\frac{(3x+1) \cdot \cos(2x-1)}{2} + \frac{3}{2} \cdot \frac{1}{2}\int \cos(2x-1) \cdot 2 \cdot dx$ =

= $-\frac{(3x+1) \cdot \cos(2x-1)}{2} + \frac{3}{4} \cdot sen(2x-1)$ + C

4) $\int \frac{Lnx}{x^2} \cdot dx$ = Lnx· $\left(-\frac{1}{x}\right)$ - $\int \left(-\frac{1}{x}\right) \cdot \frac{1}{x} \cdot dx = -\frac{Lnx}{x} + \int \frac{1}{x^2} \cdot dx = -\frac{Lnx}{x} - \frac{1}{x}$ + C = $\frac{-Lnx-1}{x}$ + C

u = Lnx ⇒ du = $1/x$·dx

dv = $\frac{1}{x^2}$·dx ⇒ v = $\int \frac{1}{x^2} \cdot dx = \int x^{-2} \cdot dx = \frac{x^{-1}}{-1} = -\frac{1}{x}$

5) $\int \frac{x}{sen^2 x} \cdot dx$ = x·(-cotgx) - $\int (-\cot gx) \cdot dx$ = -x·cotgx + $\int \frac{\cos x}{senx} \cdot dx$ = -x·cotgx + Ln| senx | + C

u = x ⇒ du = dx

dv = $\frac{1}{sen^2 x}$·dx ⇒ v = $\int \frac{1}{sen^2 x} \cdot dx$ = -cotg x

6) $\int \arccos x \cdot dx$ = x·arccosx - $\int x \cdot \frac{-1}{\sqrt{1-x^2}} \cdot dx$ = x·arccosx - $\frac{1}{2} \cdot \int -2x \cdot (1-x^2)^{-1/2} \cdot dx$ = (*)

u = arccosx ⇒ du = $\frac{-1}{\sqrt{1-x^2}}$·dx

dv = dx ⇒ v = x

(*) = x·arccosx - $\frac{1}{2}\cdot\frac{(1-x^2)^{1/2}}{1/2}$ + C = x·arccosx - $\sqrt{1-x^2}$ + C

7) $\int x\cdot\sqrt{2+x}\cdot dx$ = x·$\frac{2\cdot(2+x)^{3/2}}{3}$ - $\int\frac{2\cdot(2+x)^{3/2}}{3}\cdot dx$ = $\frac{2x\cdot\sqrt{(2+x)^3}}{3}$ - $\frac{2}{3}\cdot\int(2+x)^{3/2}\cdot dx$ = (*)

u = x ⇒ du = dx

dv = $\sqrt{2+x}$·dx ⇒ v = $\int(2+x)^{1/2}\cdot dx$ = $\frac{(2+x)^{3/2}}{3/2}$ = $\frac{2\cdot(2+x)^{3/2}}{3}$

(*) = $\frac{2x\cdot\sqrt{(2+x)^3}}{3}$ - $\frac{2}{3}\cdot\frac{(2+x)^{5/2}}{5/2}$ + C = $\frac{2x\cdot\sqrt{(2+x)^3}}{3}$ - $\frac{4\cdot\sqrt{(2+x)^5}}{15}$ + C

8) $\int x\cdot e^{3x-1}\cdot dx$ = x·$\frac{1}{3}\cdot e^{3x-1}$ - $\int\frac{1}{3}\cdot e^{3x-1}\cdot dx$ = $\frac{x\cdot e^{3x-1}}{3}$ - $\frac{1}{3}\cdot\frac{1}{3}\int e^{3x-1}\cdot 3\cdot dx$ = $\frac{x\cdot e^{3x-1}}{3}$ - $\frac{e^{3x-1}}{9}$ + C

u = x ⇒ du = dx

dv = e^{2x-1}·dx ⇒ v = $\int e^{3x-1}\cdot dx$ = $\frac{1}{3}\cdot\int e^{3x-1}\cdot 3\cdot dx$ = $\frac{1}{3}\cdot e^{3x-1}$

C.- Calcular las siguientes integrales racionales:

1) $\int\frac{x+3}{x^2-1}\cdot dx$=$\int\frac{2}{x-1}\cdot dx$ + $\int\frac{-1}{x+1}\cdot dx$= $2\cdot\int\frac{1}{x-1}\cdot dx$ - $\int\frac{1}{x+1}\cdot dx$= $2\cdot Ln|x-1|$ - $Ln|x+1|$ + C

Racional **tipo a**

$\frac{x+3}{(x-1)\cdot(x+1)}=\frac{A}{x-1}+\frac{B}{x+1}=\frac{A\cdot(x+1)+B(x-1)}{(x-1)\cdot(x+1)}$ ⇒ **x + 3 = A·(x + 1) + B·(x – 1)**

Para x = 1 ⇒ 4 = 2A + 0 ⇒ **A = 4/2 = 2**; Para x = -1 ⇒ 2 = 0 – 2B ⇒ **B = - 2/2 = - 1**

2) $\int\frac{x^2+1}{x^2+x}\cdot dx$ = $\int 1\cdot dx$ + $\int\frac{-x+1}{x^2+x}\cdot dx$ = **x + Ln|x| – 2·Ln|x+1| + C**

(*)

Racional **tipo b** ⇒ hay que dividir primero:

$$\begin{array}{rr|l} x^2 & +1 & x^2+x \\ \hline -x^2-x & & 1 \\ -x & +1 & \end{array}$$

(*) $\int\frac{-x+1}{x^2+x}\cdot dx$ = $\int\frac{1}{x}\cdot dx$ + $\int\frac{-2}{x+1}\cdot dx$ = Ln|x| – 2·Ln|x+1| + C

Racional **tipo a**

$\frac{-x+1}{x\cdot(x+1)}=\frac{A}{x}+\frac{B}{x+1}=\frac{A\cdot(x+1)+Bx}{x\cdot(x+1)}$ ⇒ - x + 1 = A·(x + 1) + B·x

Para x = 0 ⇒ 1 = A + 0 ⇒ **A = 1**; Para x = -1 ⇒ 2 = 0 – B ⇒ **B = - 2**

3) $\int \frac{5x^2-19x+12}{x^3-2x^2-5x+6}\cdot dx = \int \frac{1/3}{x-1}\cdot dx + \int \frac{14/3}{x+2}\cdot dx = \frac{1}{3}\cdot Ln|x-1| + \frac{14}{3}\cdot Ln|x+2| + C$

Racional **tipo a**

Se descompone el polinomio $x^3 - 2x^2 - 5x + 6 = (x-3)\cdot(x-1)\cdot(x+2)$ por Ruffini.

$$\frac{5x^2-19x+12}{(x-3)\cdot(x-1)\cdot(x+2)} = \frac{A}{x-3} + \frac{B}{x-1} + \frac{C}{x+2} = \frac{A\cdot(x-1)\cdot(x+2)+B(x-3)\cdot(x+2)+C(x-3)\cdot(x-1)}{(x-3)\cdot(x-1)\cdot(x+2)}$$

$\Rightarrow$ $5x^2 - 19x + 12 = A\cdot(x-1)\cdot(x+2) + B\cdot(x-3)\cdot(x+2) + C(x-3)\cdot(x-1)$

Para $x = 3 \Rightarrow 45 - 57 + 12 = 10A + 0 + 0 \Rightarrow 10A = 0 \Rightarrow$ **A = 0**

Para $x = 1 \Rightarrow 5 - 19 + 12 = 0 - 6B + 0 \Rightarrow -6B = -2 \Rightarrow B = 2/6 \Rightarrow$ **B = 1/3**

Para $x = -2 \Rightarrow 20 + 38 + 12 = 0 + 0 + 15C \Rightarrow 15C = 70 \Rightarrow C = 70/15 \Rightarrow$ **C = 14/3**

4) $\int \frac{x^3}{(x-2)^2}\cdot dx = \int (x+4)\cdot dx + \int \frac{12x-16}{(x-2)^2}\cdot dx\ (*) = \frac{x^2}{2} + 4x + 12\cdot Ln|x-1| - \frac{8}{x-2} + C$

Racional **tipo b** $\Rightarrow$ Hay que dividir:

$$\begin{array}{l|l} x^3 & x^2-4x+4 \\ \hline -x^3+4x^2-4x & x+4 \\ \quad 4x^2-4x & \\ \quad -4x^2+16x-16 & \\ \quad\quad 12x-16 & \end{array}$$

(*)$= \int \frac{12x-16}{(x-2)^2}\cdot dx = \int \frac{12}{x-2}\cdot dx + \int \frac{8}{(x-2)^2}\cdot dx = 12\cdot Ln|x-1| + 8\cdot\int (x-2)^{-2}\cdot dx =$

Racional **tipo a**

$= 12\cdot Ln|x-1| + 8\cdot\frac{(x-2)^{-1}}{-1} + C = 12\cdot Ln|x-1| - \frac{8}{x-2} + C$

$$\frac{12x-16}{(x-2)^2} = \frac{A}{x-2} + \frac{B}{(x-2)^2} = \frac{A\cdot(x-2)+B}{(x-2)^2} \Rightarrow 12x - 16 = A\cdot(x-2) + B$$

Para $x = 2 \Rightarrow 8 = B \Rightarrow$ **B = 8**; Para $x = 0 \Rightarrow -16 = -2A + 8 \Rightarrow 2A = 24 \Rightarrow$ **A = 12**

5) $\int \frac{5x+3}{x^2+9}\cdot dx = \int \frac{5x}{x^2+9}\cdot dx + \int \frac{3}{x^2+9}\cdot dx = \frac{5}{2}\cdot\int \frac{2x}{x^2+9}\cdot dx + \int \frac{3/9}{x^2/9+1}\cdot dx =$

Racional **tipo a**: $x^2 + 9 = 0 \Rightarrow x^2 = -9$ No existe

$= \frac{5}{2}\cdot Ln|x^2+9| + \int \frac{1/3}{(x/3)^2+1}\cdot dx = \frac{5}{2}\cdot Ln|x^2+9| + arctg(x/3) + C$

6) $\int \frac{x^5}{x^4-1}\cdot dx = \int x\cdot dx + \int \frac{x}{x^4-1}\cdot dx = \frac{x^2}{2} + \frac{1}{4}\cdot Ln|x-1| + \frac{1}{4}\cdot Ln|x+1| - \frac{1}{4}\cdot Ln(x^2+1) + C$

(*)

Racional **tipo b** $\Rightarrow$ hay que dividir primero:

$$\begin{array}{l|l} x^5 & x^4-1 \\ \hline -x^5+x & x \\ \quad\quad x & \end{array}$$

(*) $\int \frac{x}{x^4-1}\cdot dx = \int \frac{1/4}{x-1}\cdot dx + \int \frac{1/4}{x+1}\cdot dx + \int \frac{-1/2\, x}{x^2+1}\cdot dx = \frac{1}{4}\cdot Ln|x-1| + \frac{1}{4}\cdot Ln|x+1|$ -

Racional **tipo a**

$- \frac{1}{2}\cdot\frac{1}{2}\int \frac{2x}{x^2+1}\cdot dx = \frac{1}{4}\cdot Ln|x-1| + \frac{1}{4}\cdot Ln|x+1| - \frac{1}{4}\cdot Ln(x^2+1) + C$

Se descompone el polinomio $x^4 - 1 = (x+1)\cdot(x-1)\cdot(x^2+1)$ por Ruffini.

$$\frac{x}{(x-1)\cdot(x+1)\cdot(x^2+1)} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{Cx+D}{x^2+1}$$

$$\frac{x}{(x-1)\cdot(x+1)\cdot(x^2+1)} = \frac{A\cdot(x+1)\cdot(x^2+1) + B\cdot(x-1)\cdot(x^2+1) + Cx\cdot(x-1)\cdot(x+1)\cdot + D\cdot(x-1)\cdot(x+1)}{(x-1)\cdot(x+1)\cdot(x^2+1)}$$

$x = A\cdot(x+1)\cdot(x^2+1) + B\cdot(x-1)\cdot(x^2+1) + Cx\cdot(x-1)\cdot(x+1)\cdot + D\cdot(x-1)\cdot(x+1)$

Para x = 1 ⇒ 1 = 4A ⇒ **A = ¼;** Para x = - 1 ⇒ -1 = - 4B ⇒ **B = 1/4**

Para x = 0 ⇒ 0 = ¼ - ¼ - D ⇒ **D = 0**; Para x = 2 ⇒ 2 =15/4 + 5/4 + 6C ⇒ **C = - 3/6 = -1/2**

7) $\int \frac{2}{x^3+3x}\cdot dx = \int \frac{2/3}{x}\cdot dx + \int \frac{-2/3\, x}{x^2+3}\cdot dx = \frac{2}{3}\cdot Ln|x| - \frac{2}{3}\cdot\frac{1}{2}\cdot\int \frac{2x}{x^2+3}\cdot dx = (*)$

Racional **tipo a:** $x^3 + 3x = 0 \Rightarrow x\cdot(x^2+3) = 0 \Rightarrow x = 0$ ó $x^2 + 3 = 0 \Rightarrow x = \sqrt{-3}$ No Existe

$$\frac{2}{x\cdot(x^2+3)} = \frac{A}{x} + \frac{Bx+C}{x^2+3} = \frac{A\cdot(x^2+3) + B\cdot x^2 + C\cdot x}{x\cdot(x^2+3)} \Rightarrow 2 = A\cdot(x^2+3) + B\cdot x^2 + C\cdot x$$

Para x = 0 ⇒ 2 = 3A ⇒ **A = 2/3;** Para x = 1 ⇒ 2 = 8/3 + B + C ⇒ **B + C = - 2/3**
Para x = -1 ⇒ 2 = 8/3 + B – C ⇒ **B – C = - 2/3**

$\begin{cases} B + C = -2/3 \\ B - C = -2/3 \end{cases}$ ⇒ 2B = -4/3 = ⇒ **B = - 2/3** ⇒ - 2/3 + C = -2/3 ⇒ **C = 0**

$(*) = \frac{2}{3}\cdot Ln|x| - \frac{1}{3}\cdot Ln|x^2+3| + C$

D.- Calcular las siguientes integrales, por sustitución o cambio de variable:

1) $\int \frac{senx}{1+\cos^2 x}\cdot dx = -\int \frac{-senx}{1+\cos^2 x}\cdot dx = -\int \frac{1}{1+t^2}\cdot dt$ = - arctg(t) + C = - arctg(cosx) + C

t = cos x ⇒ dt = - senx·dx

2) $\int \frac{\cos x}{sen^2 x - senx}\cdot dx = \; = \int \frac{1}{t^2 - t}\cdot dt \; = \int \frac{-1}{t}\cdot dt + \int \frac{1}{t-1}\cdot dt$ =(*)

t = sen x $\Rightarrow$ dt = cosx·dx   racional tipo a

$\frac{1}{t\cdot(t-1)} = \frac{A}{t} + \frac{B}{t-1} = \frac{A\cdot(t-1)+B\cdot t}{t\cdot(t-1)}$ $\Rightarrow$ 1 = A·(t – 1) + B·t

Para t = 0 $\Rightarrow$ 1 = -A + 0 $\Rightarrow$ **A = -1** ; Para t = 1 $\Rightarrow$ 1 = B $\Rightarrow$ **B = 1**

(*) = - Ln|t| + Ln| t – 1| + C = **- Ln| senx | + Ln| senx – 1| + C**

3) $\int sen^3 x\cdot dx = \int sen^2 x\cdot senx\cdot dx = -\int sen^2 x\cdot(-senx)\cdot dx = -\int(1-t^2)\cdot dt$ = - t + $\frac{t^3}{3}$ + C = -cosx + $\frac{\cos^3 x}{3}$ + C

t = cos x $\Rightarrow$ dt = - senx·dx ; **$sen^2x = 1 - cos^2x = 1 - t^2$**

4) $\int \frac{\cos^3 x}{sen^4 x}\cdot dx = \int \frac{\cos^2 x\cdot\cos x}{sen^2 x\cdot sen^2 x}\cdot dx = \int \frac{1-t^2}{t^2\cdot t^2}\cdot dt \; = \int \frac{1}{t^4}\cdot dt - \int \frac{t^2}{t^4}\cdot dt =$

t = sen x $\Rightarrow$ dt = cosx·dx; **$cos^2x = 1 - sen^2x = 1 - t^2$**

$= \int t^{-4}\cdot dt - \int t^{-2}\cdot dt = \frac{t^{-3}}{-3} - \frac{t^{-1}}{-1} + C = \frac{-1}{3\cdot t^3} + \frac{1}{t} + C = \frac{-1}{3\cdot sen^3 x} + \frac{1}{senx} + C$

5) $\int \frac{1}{x+1+\sqrt{x+1}}\cdot dx = \int \frac{1}{t^2+\sqrt{t^2}}\cdot 2t\cdot dt = \int \frac{2t}{t^2+t}\cdot dt = \int \frac{2}{t+1}\cdot dt$ =2·Ln|t+1| + C = $2\cdot Ln\left|\sqrt{x+1}+1\right| + C$

Cambio: $t^2 = x + 1 \Rightarrow 2t\cdot dt = dx$

6) $\int \frac{x+2}{1+\sqrt{x+2}}\cdot dx = \int \frac{t^2}{1+\sqrt{t^2}}\cdot 2t\cdot dt = \int \frac{2t^3}{1+t}\cdot dt = \int(2t^2-2t+2)dt + \int \frac{-2}{t+1}\cdot dt$ =(*)

Cambio: $t^2 = x + 2 \Rightarrow 2t\cdot dt = dx$ Integral racional tipo **b**

```
  2t^3                | t + 1
- 2t^3 – 2t^2         2t^2 – 2t + 2
       – 2t^2
         2t^2 + 2t
              + 2t
            – 2t – 2
              - 2
```

(*) = $2\cdot\frac{t^3}{3} - t^2 + 2t$ – 2·Ln|t + 1| + C = $\frac{2\cdot\sqrt{(x+2)^3}}{3} - (x+2) + 2\sqrt{x+2} - 2\cdot Ln\left|\sqrt{x+2}+1\right| + C$