SOLUCIONES: EXAMEN DE INTEGRALES

1.- Calcular las siguientes integrales:

a) $\int \frac{3x-\sqrt[3]{x}}{x^2}\cdot dx = \int \frac{3x}{x^2}\cdot dx - \int \frac{\sqrt[3]{x}}{x^2}\cdot dx = \int \frac{3}{x}\cdot dx - \int \frac{x^{1/3}}{x^2}\cdot dx =$ 3·Ln|x| $-\int x^{-5/3}\cdot dx =$

= 3·Ln|x| $-\frac{x^{-2/3}}{-2/3}+C =$ 3·Ln|x| $+\frac{3}{2\cdot\sqrt[3]{x^2}}+C$

b) $\int \cos^3 x\cdot dx = \int \cos^2 x\cdot \cos x\cdot dx = \int (1-t^2)\cdot dt = t-\frac{t^3}{3}+C = senx-\frac{sen^3 x}{3}+C$

Cambio: **t = sen x** $\Rightarrow$ **dt = cos x·dx** $\Rightarrow$ $\cos^2 x = 1 - sen^2 x =$ **$1 - t^2$**

c) $\int (4x^2+2)\cdot sen2x\cdot dx = (4x^2+2)\cdot\left(-\frac{\cos 2x}{2}\right) - \int 8x\cdot\left(-\frac{\cos 2x}{2}\right)\cdot dx = -\frac{(4x^2+2)\cdot\cos 2x}{2} + \int 4x\cdot\cos 2x\cdot dx =$

$u = 4x^2 + 2 \Rightarrow du = 8x\cdot dx$	$u = 4x \Rightarrow du = 4\cdot dx$
$dv = sen2x\cdot dx \Rightarrow v = \frac{-\cos 2x}{2}$	$dv = \cos 2x\cdot dx \Rightarrow v = \frac{sen2x}{2}$

(*) = $-\frac{(4x^2+2)\cdot\cos 2x}{2} + \left[4x\cdot\frac{sen2x}{2} - \int 4\cdot\frac{sen2x}{2}\cdot dx\right] = -\frac{(4x^2+2)\cdot\cos 2x}{2} + 2x\cdot sen2x + \cos 2x + C$

d) $\int \frac{3x^2-4x-1}{x^2-2x+1}\cdot dx = \int 3\cdot dx + \int \frac{2x-4}{x^2-2x+1}\cdot dx = 3x + \int \frac{2}{x-1}\cdot dx + \int \frac{-2}{(x-1)^2}\cdot dx =$ (*)

racional tipo b

$3x^2 - 4x - 1$	$x^2 - 2x + 1$
$-3x^2 + 6x - 3$	
$2x - 4$	3

$x^2 - 2x + 1 = 0 \Rightarrow x = 1$ doble

$\frac{2x-4}{(x-1)^2} = \frac{A}{x-1} + \frac{B}{(x-1)^2} = \frac{A(x-1)+B}{(x-1)^2}$ $\Rightarrow$ **2x – 4 = A·(x – 1) + B**

Si x = 1 $\Rightarrow$ **- 2 = B** ; Si x = 0 $\Rightarrow$ - 4 = - A – 2 $\Rightarrow$ **A = 2**

(*) =3x + 2·Ln|x – 1| – $2\cdot\int (x-1)^{-2}\cdot dx =$ 3x + 2·Ln|x – 1| – $2\cdot\frac{(x-1)^{-1}}{-1}+C$ = 3x + 2·Ln|x – 1| $+\frac{2}{x-1}+C$

e) $\int \frac{e^{\sqrt{x}}}{5\cdot\sqrt{x}}\cdot dx = \int e^{\sqrt{x}}\cdot\frac{1}{5\cdot\sqrt{x}}\cdot dx = \frac{2}{5}\int e^{\sqrt{x}}\cdot\frac{1}{2\cdot\sqrt{x}}\cdot dx = \frac{2}{5}\cdot e^{\sqrt{x}} + C$

f) $\int \frac{1}{x\cdot\sqrt{x+4}}\cdot dx = \int \frac{1}{(t^2-4)\cdot t}\cdot 2t\cdot dt = \int \frac{2}{(t^2-4)}\cdot dt = \int \frac{1/2}{t-2}\cdot dt + \int \frac{-1/2}{t+2}\cdot dt =$

$\frac{1}{2}\cdot Ln|t-2| - \frac{1}{2}\cdot Ln|t+2| + C =$(*)

Cambio: $t^2 = x + 4 \Rightarrow 2t\cdot dt = dx$

$\frac{2}{(t-2)\cdot(t+2)} = \frac{A}{t-2} + \frac{B}{t+2} = \frac{A\cdot(t+2)+B(t-2)}{(t-2)\cdot(t+2)} \Rightarrow 2 = A\cdot(t + 2) + B\cdot(t - 2) \Rightarrow$

$\Rightarrow$ Si $t = 2 \Rightarrow 2 = 4A \Rightarrow$ **A = 1/2**

Si $t = -2 \Rightarrow 2 = -4B \Rightarrow$ **B = -1/2**

(*)$= \frac{1}{2}\cdot Ln\left|\sqrt{x+4}-2\right| - \frac{1}{2}\cdot Ln\left|\sqrt{x+4}+2\right| + C$

g) $\int \frac{3x^2+x+2}{x^3-2x^2+4x-8}\cdot dx = \int \frac{2}{x-2}\cdot dx + \int \frac{x+3}{x^2+4}\cdot dx = 2\cdot Ln|x-2| + \int \frac{x}{x^2+4}\cdot dx + \int \frac{3}{x^2+4}\cdot dx =$ (*)

	1	-2	4	-8
2		2	0	8
	1	0	4	0

$x^2 + 4 = 0 \Rightarrow x^2 = -4$ no tiene solución

$\frac{3x^2+x+2}{(x-2)\cdot(x^2+4)} = \frac{A}{x-2} + \frac{Bx+C}{x^2+4} = \frac{A\cdot(x^2+4)+B\cdot x\cdot(x-2)+C\cdot(x-2)}{(x-2)\cdot(x^2+4)} \Rightarrow$

$\Rightarrow$ **$3x^2 + x + 2 = A\cdot(x^2 + 4) + B\cdot x\cdot(x - 2) + C\cdot(x - 2)$**

Si $x = 2 \Rightarrow 16 = 8\cdot A \Rightarrow$ **A = 2**; Si $x = 0 \Rightarrow 2 = 8 - 2C \Rightarrow$ **C = 3**

Si $x = 1 \Rightarrow 6 = 10 - B - 3 \Rightarrow$ **B = 1**

(*)$= 2\cdot Ln|x-2| + \frac{1}{2}\cdot\int \frac{2x}{x^2+4}\cdot dx + \int \frac{3/4}{x^2/4+1}\cdot dx = 2\cdot Ln|x-2| + \frac{1}{2}\cdot Ln|x^2+4| + \frac{3}{4}\cdot 2\int \frac{1/2}{(x/2)^2+1}\cdot dx =$

$= 2\cdot Ln|x-2| + \frac{1}{2}\cdot Ln|x^2+4| + \frac{3}{2}\cdot arctg\left(\frac{x}{2}\right) + C$

2.- Si $f'(x) = \frac{Lnx}{x^3}$, hallar f(x) sabiendo que $f(1) = \frac{1}{2}$.

$f(x) = \int \frac{Lnx}{x^3}\cdot dx = Lnx\cdot\left(-\frac{1}{2x^2}\right) - \int\left(-\frac{1}{2x^2}\right)\cdot\frac{1}{x}\cdot dx = -\frac{Lnx}{2x^2} + \frac{1}{2}\cdot\int\frac{1}{x^3}\cdot dx = -\frac{Lnx}{2x^2} - \frac{1}{4x^2} + C$

$u = Lnx \Rightarrow du = 1/x\cdot dx$

$dv = \frac{1}{x^3}\cdot dx \Rightarrow v = \int \frac{1}{x^3}\cdot dx = \int x^{-3}\cdot dx = \frac{x^{-2}}{-2} = -\frac{1}{2x^2}$

$f(1) = -\frac{Ln1}{1} - \frac{1}{4} + C = \frac{1}{2} \Rightarrow C = 3/4 \Rightarrow f(x) = -\frac{Lnx}{2x^2} - \frac{1}{4x^2} + \frac{3}{4}$